THE MATHEMATICS TEACHER

VOLUME XIV OCTOBER 1921 NUMBER 6

THE AIMS OF MATHEMATICAL EDUCATION

By PROFESSOR J. H. MINNICK
Dean of School of Education, University of Pennsylvania

Education is a complex process involving a variety of experiences gained through both school and out-of-school activities. Each subject of the curriculum should make its definite contribution to this experience, but we must be sure that the result is a unit. An investigation of conditions in most of our high schools will show that a child is under the instruction of perhaps four or five teachers, all of whom are working independently of each other. Very seldom does one teacher know what the others are trying to do. In order to avoid such conditions and to insure a unified education for each individual, it is necessary that the aim of each subject shall be determined in the light of the general definition of education. Only by this means can the subject matter of each course be so selected and presented that there is neither *useless* overlapping on the one hand nor the omission of important elements on the other hand. Hence, in discussing the aim of mathematical education, we should consider the general meaning of education and then determine what contribution mathematics can make most effectively. For this purpose we shall accept Ruediger's definition, namely, ". . . to educate a person means to adjust him to those elements of his environment that are of concern in modern life, and to develop, organize, and train his powers so that he may make efficient and proper use of them."[1] This definition consists of two parts. One of these is concerned with the adjustment of the individual to his environment; this is the objective side. The other is concerned with the development of the powers of the individual;

[1] Ruediger: The Principles of Education, p. 39.

this is the subjective side of education. However, one's powers are developed only by contact with and adjustment to his environment, and he is adjusted to his environment only through his powers and abilities. Thus, a child's power to think correctly is developed most effectively when he is brought face to face with a real situation the solution of which is vital to his welfare; but he can successfully master the situation only by the use of his reasoning power or such other abilities as may be involved. Hence, the two parts of this definition are not independent and we need not consider them separately; when one is satisfied in the most effective way the other will be. At present we shall confine our attention to the objective phase of education.

The environment to which we must be adjusted may be thought of as consisting of two elements; namely the material and the social. Independently of his social relationship each individual is dependent upon and interested in the material universe in which he lives. This universe constitutes man's material environment. But each individual under normal conditions also has relationship with other men. That portion of the human race with which one has any such relationship constitutes his social environment. However, these two elements of our environment are not independent. In order to maintain our social relationship we have been forced to lay hold of elements of the material world and adapt them to our use. We build houses, construct bridges, cut down forests and tunnel mountains. Our question is, In what way does mathematics aid in adjusting each individual to his material and social environment? We shall first consider the material, and later the social environment.

From the earliest period man has struggled, not only to lay hold of and use the material things round about him, but also to understand and appreciate the natural phenomena which he sees daily. Mythology and the beginnings of science are largely the results of this natural curiosity on the part of man. As is well known to every scientist, the material world in which we live is subject to mathematical laws; the path of every heavenly body, the force of gravity, and the properties of electricity are mathematical in their nature. Much of the early mathematics was discovered in an attempt to interpret the material universe,

and today one of the most effective means of understanding the mysteries of nature is mathematics. In the high school it is not possible to give a pupil such a knowledge of mathematics as will enable him to compute the path of a comet or to determine the exact time of an eclipse. However, even a knowledge of very elementary mathematics will enable the child to measure inaccessible distances and solve other seemingly impossible problems. This should increase his faith in mathematics as a means of solving the more difficult problems of nature and enable him to accept the conclusions of expert mathematicians. By the solution of those simple problems and citing more difficult applications *we should give each individual such a knowledge of the subject as will enable him to understand the exactness and force with which mathematics works and the parts which it plays in solving the problems of nature.*

So far we have considered only man's attempt to satisfy his curiosity in the natural phenomena. But he has also undertaken to master nature and turn it to his own use. His success has depended largely upon his ability to apply mathematics in some form to the situations confronting him. Problems relating to lighting and heating, construction of buildings, and the control of electricity are mathematical in nature. It is true that these problems are handed over to a few experts and the majority of people never have to solve them. Nevertheless we are all dependent upon the results, and we will accept them more readily if we have some intelligent understanding of the process by which they have been obtained. We cannot hope to give each child a knowledge of mathematics which will enable him to solve all the problems upon which his life depends. However, by properly selecting the subject matter of the mathematics course and presenting it in the right relationships *we should endeavor to give to each child such a knowledge of mathematics as will enable him to understand the exactness and force of the subject which makes it possible for man to turn the elements of nature to his own use.*

One of the most important elements in maintaining our social relations and therefore in adjusting us to our social environment is our vocabulary. When discussing the development of the vocabulary we must not make too large a claim for mathe-

matics. Nevertheless, mathematical principles and concepts, perhaps enter into our daily conversation and reading to a greater extent than most of us think. The more popular technical magazines contain mathematical discussions of topics of common interest. Even such magazines as the *Literary Digest* often contain discussions involving statistics and other forms of mathematics. In our daily conversation such mathematical terms as radius, circular, equal, negative, positive, etc., are commonly used. We should not attempt to justify mathematics on the basis of its contribution to vocabularly control; but having justified it on some other basis, *we should, insofar as is consistent with the more distinctive aims of mathematical education, develop such fundamental concepts as will enable the child to express his thoughts more clearly and to understand written and spoken language more readily.*

Adjustment to the social environment also means the best possible preparation to render effective service to society. Mathematical education plays an important part in this preparation in several ways. Every occupation requires some knowledge of mathematics. The country grocer uses only the simplest arithmetic, while leaders in finances, engineering and sciences require an extensive knowledge of advanced mathematics; the farmer uses geometry (learned through experience) in the construction of buildings, bins and fences; and the shop man often has need of both geometry and the formula. In fact, as Dr. David Eugene Smith[1] suggests, the removal of all knowledge of mathematics from the world would mean that most, if not all, occupations would be impossible. Therefore, *we should, as nearly as possible, give to each child such a knowledge of mathematics as will enable him to carry on the work of his future occupation as it is now conducted.*

However, it is not possible to foretell the exact destination of each child passing through our schools and it is not possible to make adequate preparation for all possible lines of activity that may be entered upon by every pupil in our classes. During the past year at least seven able young people have been brought to our attention, who had not intended going to college until

[1] David Eugene Smith: Mathematics in the training for citizenship. Teachers' College Record. May, 1917.

near the close of their high school course. When the opportunity for a college education was offered they found that the high school not only had not given them the mathematics necessary for college entrance but it had not grounded them in the simple fundamentals which would enable them to make the necessary preparation. Also those who have entered upon one work are frequently transferred to another or are promoted into advanced positions which require additional preparation. A young man from the shops of Wilkes-Barre, Pa., asked me to help him select a text in trigonometry. During the conversation it appeared that he had reached the place in his trade where further promotion was impossible until he had mastered certain parts of trigonometry. The tragic part was that he did not have a knowledge of those simple algebraic and geometric facts which serve as a basis for reading his trigonometry. Time after time our attention has been called to such cases. In order that such people may meet the new conditions successfully, *we should give to each child such knowledge of mathematics as will serve as a basis for future preparation, if progress in his work should demand it.*

In many of our commercial and industrial concerns the work is being carried on today as it was years ago, because the employees have had slight educational advantages and the specific training for their work has been in the institution where they now work or in a similar institution. In a certain Philadelphia metal shop the foreman learned to do, in a mechanical way, the work as his master did it a generation ago and his son is now being trained in the same way. Consequently the discovery and introduction of new methods is almost impossible. In a New York bank the president was required to puzzle out conclusions from long lists of figures until a recent high school graduate was employed who sent in his report with a graphical representation of such lists of figures. From that day the graphical method of making reports was used in that bank. On the other hand those who have had mathematical training frequently have not had practical experience and therefore do not know where mathematics can be applied. Not long since a professor of mathematics in one of our leading universities said that we had as well admit that our subject has no practical values.

Nevertheless, his special subject solves problems of sanitation, irrigation and electricity. Progress is possible only when those who have had practical experience have also had such mathematical training as will enable them to see where and how improvement can be made. Therefore, *we should, as nearly as possible, give to each child such a knowledge of mathematics as will enable him to find new and better ways of doing his work.*

After schools have done all that is possible to meet the needs of individuals, each will sometimes face mathematical situations which he cannot solve. It is important that, in all such cases, he shall be able to recognize the situation as mathematical and, if he cannot solve it, refer it to a specialist. Insurance companies and many other business and industrial establishments employ their own mathematical experts to whom their problems are referred. However, individuals often do not know that their situations are mathematical and are therefore unable to call upon an expert for assistance. The head of the household arts department in a well known university declared that the only mathematics needed by the students of her department was a very little simple arithmetic. Not long afterward an instructor in her department asked each student in her class to buy one and one-half yards of expensive material to make a hat which could easily be made from one yard of material. Not knowing that the problem was capable of solution she guessed at the amount, although any one of fifteen or twenty mathematicians connected with the institution could have solved the problem for her and saved each member of her class an extra expense of two dollars. During the war two nurses of New York, prompted by the best of motives, resorted to the chain-letter method of raising money for war orphans. Not recognizing in the chain-letter a problem capable of solution they set the number of letters in the chain so high that there were not enough people in the United States to send it to. In neither of these cases can we blame those concerned for not being able to solve their problems, but they should have recognized them as mathematical in nature and sought the aid of the specialist. We cannot hope to give each pupil enough mathematics to enable him to solve all his problems: but *we should give him such a knowledge of the subject as will enable him to recognize a mathe-*

matical situation when he meets it, and if he cannot solve it he can take it to an expert.

One of the most important functions of the school is to discover the abilities and aptitudes of children and thus help each find the place where he will make the greatest possible contribution to society. Out of the masses of children now in school there are a few who will develop into leaders in mathematical research. The value of their work to society cannot be overestimated. They will discover new mathematical principles, the applications of which will give man a more complete mastery of his environment. Since it is only by contact with a subject that special ability along a given line can be determined, *we should require of each pupil sufficient mathematics to determine whether he will profit by further study of the subject and to select those who will probably be leaders in mathematical thought.*

If those who show marked ability are to develop into leaders and make their fullest contribution to society, the school must make it possible for them to advance as rapidly as possible. Therefore, *for this select group we should give enough mathematics to keep the door to specialization in mathematics and in fields dependent upon mathematics open.*

As previously stated the subjective or development aim of education is best realized when the objective or adjustment aim is satisfied in the most effective way.[1] Mathematicians have, for the most part, justified their subject on the plea that it is the best possible training in reasoning.[2] However, if we accept Dewey's analysis of the thought process,[3] we are compelled to conclude that, as presented in many of the high schools, the study of mathematics involves almost none of the elements of thought. On the other hand, if the subject is taught in a practical way so as to realize the several aims stated above, all the elements of thought are involved. Thus, if workmen have staked off the foundation of a house there may be a feeling that the work is inaccurate. This vague feeling is developed into a definite question, "How can we test a four-sided figure to see if it

1 David Mair: School Course in Mathematics. p. iii.
2 David Eugene Smith: The Teaching of Elementary Mathematics, p. 238.
3 John Dewey: How We Think. pp. 9-13.

is a rectangle?" In the light of known facts they formulate a tentative solution, namely, "the opposite sides must be equal." Upon further investigation they find that this is not sufficient. They then revise their probable solution by adding "and the diagonals must be equal." A further investigation shows this revised solution to be correct and they proceed to check their work.

This type of teaching has several advantages. First, it shows the child the usefulness of the mathematical facts which he is studying, and in the future he will be more likely to recognize possible applications. Second, the investigation of his tentative solutions involves just as thorough a mastery of geometry as the slavish followings of a textbook does. Third, since it involves all the steps of the thought process the disciplinary value will be realized in the fullest possible way. Thorndike says,[1] "Teach nothing merely because of its disciplinary value, but teach everything so as to get what disciplinary value it has." Fortunately, in mathematics the best way to get "what disciplinary value it has" is to get the other values in the most effective way.

In order that no one may assume that we have[*] in mind an extensive course in secondary mathematics, it should be said, in conclusion, that the realization of the above aims will require much less time than is now given to mathematics, but it will require a more carefully selected content and a different method of presentation.

[1] E. L. Thorndike: Principles of Teaching, p. 249.
[*] Read before the Association of Mathematics Teachers of New Jersey at regular meetings in November, 1919, and in October, 1920.

EMPIRICAL RESULTS IN THE THEORY OF NUMBERS

By PROFESSOR R. D. CARMICHAEL
University of Illinois

The subject of prime numbers is one of exceeding difficulty. There are some remarkable results which are easily proved. Euclid gave a very simple demonstration that the number of primes is infinite. If the number of primes were finite and their product were denoted by M, then $M + 1$ would be a number without a prime factor; hence the number of primes is infinite. It is easy to propose problems about prime numbers which no one has been able to solve though numerous investigators have had them in consideration. The following are some such problems having a simple formulation:

1. Is there an infinite number of pairs of primes differing by 2?
2. Is every even number the sum of two primes?
3. Is every even number the difference of two primes?
4. To find a prime number greater than a given prime.
5. To find the prime number which follows a given prime.
6. To find the number of primes not greater than a given number.
7. To compute directly the nth prime number, when n is given.

It has been conjectured that the true answer to each of the first three of these is affirmative. The solution of the fifth evidently involves that of the fourth. The sixth and seventh are probably still more difficult than the fifth. That the fourth is unsolved implies that we have no means yet at hand for isolating an infinite sequence of primes. While it is known that the number of primes is infinite the set of primes which have been isolated and recognized each as a prime is finite. Among the known large primes may be mentioned the following:

$$2^{75} \cdot 5 + 1,\ 2^{89} - 1,\ 2^{107} - 1,\ 2^{127} - 1.$$

Fermat believed that $2^n + 1$ is always prime when n is a power of 2. Euler proved that this is incorrect by showing that

$$2^{32} + 1 = 641 \cdot 6700417.$$

It has been conjectured that the infinite sequence of numbers

$$2+1,\quad 2^2+1,\quad 2^{2^2}+1,\quad 2^{2^{2^2}}+1,\quad \dots$$

contains only prime numbers; this conjecture has been neither proved nor disproved. A demonstration of it, if it is true, would give us a new sort of theorem in the theory of numbers. The same character of result would be in evidence if one could prove the following unverified conjecture: If $2^n - 1$ is a prime p, then $2^p - 1$ is a prime. Thus $2^2 - 1 = 3$, a prime; $2^3 - 1 = 7$, a prime; $2^7 - 1 = 127$, a prime; $2^{127} - 1 =$ a prime. It has been conjectured that the sequence continues to run in primes; but the fact is not determined beyond the point indicated. Again, we have $2^5 - 1 = 31$, a prime; $2^{31} - 1 =$ a prime; but again it is unknown whether the sequence continues in prime numbers, though it has been conjectured that it does.

On the other hand it has been conjectured by Gerardin that if p is any number and a any divisor of $2^p - 1$ of the form $8m \pm 1$ and not of the form $2^n - 1$, then $2^a - 1$ is composite. (See Dickson's *History of the Theory of Numbers*, Vol. I, p. 30). It seems not to have been observed heretofore that this conjecture is false. If we take $p = 11$ we have $2^p - 1 = 2^{11} - 1 = 23 \cdot 89$, so that 89 is an admissible value of a. But $2^{89} - 1$ is known to be prime (Dickson, l. c., p. 30). Hence the conjecture of Gerardin is false. A similar conjecture by Tarry is the following: If p is a prime such that $2^p - 1$ is composite and a is the least factor (> 1) of $2^p - 1$, then $2^a - 1$ is composite. This has been verified by Tarry for the known composite numbers $2^p - 1$.

Some other conjectured theorems about prime numbers recorded in volume I of Dickson's *History* are the following:

1. If n is a prime of the form $24x + 11$ [of the form $24x + 23$] and if $2^n - 1$ is composite, the least factor of $2^n - 1$ is of the form $24y + 23$ [of the form $48y + 47$] (pp. 29, 30, 31). (It is difficult to have much confidence in conjectures of this sort.)

2. If $n > 1$ there is at least one prime in each of the intervals $n(n-1)$ to n^2, n^2 to $n(n+1)$ (p. 435). (This is a theorem of great importance, if true; and a demonstration of it would be received with pleasure by a considerable number of investigators.)

3. At least four primes lie between the squares of two consecutive primes each greater than 3 (p. 436). (This, again, is a theorem of importance.)

4. The $(2m+1)^{\text{th}}$ prime in order of magnitude (unity being counted as a prime) can be composed by addition and subtraction of all the smaller primes each taken once; the $(2m)^{\text{th}}$ prime can be composed similarly, except that the next earlier prime is doubled (p. 436).

It is known that $mf + n$, for the varying integer f, represents infinitely many primes if m and n are relatively prime (see Dickson, l. c., pp. 415-417). Let us consider the extension of this problem to the case of m arithmetical progressions $a_i n + b_i$, $i = 1, 2, \ldots, m$, where a_i and b_i are given numbers and n runs over the set 1, 2, 3, . . . Do there exist m such linear forms which give m prime numbers for the same value of n, where n runs through a determined infinite succession of positive integers? This question has been considered by L. E. Dickson (*Messenger of Mathematics*, 33 [1904]: 155-161). He finds the following necessary condition: In order that m forms $a_i n + b_i$ shall give m prime numbers for at least one integer n, it is necessary, for every prime $p \leqq m$ and for every set of p of the a_i chosen from those not divisible by p, that at least two of the b_i/a_i shall be congruent modulo p*. Dickson then inquires whether these conditions are sufficient to insure that the $a_i n + b_i$ shall represent an infinitude of sets of m primes.

Another conjectured generalization of the theorem concerning the infinitude of primes in an arithmetical progression has been stated in the following way (See Dickson, l. c., p. 333): Let N be the greatest common divisor of all integers represented by a polynomial $f(x)$ with integral coefficients without a common factor; it is conjectured that $f(x)/N$ represents an infinitude of primes when $f(x)$ is irreducible, where x ranges over the integers 1, 2, 3, . . .

It appears that this (conjectured) theorem may be capable of the following generalization: Let $f(x)$ be an irreducible polynomial with rational coefficients such that the numerical value of $f(x)$ is integral for every positive integral value of x; let N

* Two numbers are said to be **congruent** modulo p if their difference is divisible by p.

be the greatest common divisor of all the integers $f(1)$, $f(2)$, $f(3)$, . . .; then it appears probable that $f(x)/N$ has an infinitude of prime values when x ranges over the set 1, 2, 3, ... If we have $f(x) = x^3 + \frac{1}{2}x^2 + \frac{1}{2}x + 5$ we have $f(1) = 7$, $f(2) = 16$, so that $N = 1$ in this case. Applied to this case the conjecture is that $f(x)$ has a prime value for an infinite number of values of x.

It would, of course, be a matter of great interest to have a proof of either one of the two foregoing general theorems; but it seems certain that the proof is a matter of great difficulty. For the history of certain related matters the reader may consult Dickson, l. c., pp. 417-418.

It was conjectured by R. Murphy in 1841 (see Dickson, l. c., p. 186) that every prime $an^2 + p$ has a as a primitive root* if $p > a/2$, p is a prime less than a, and if a is a primitive root of p. For example, a prime $10n^2 + 7$ has 10 as a primitive root.

A. Cunningham (see Dickson, l. c., p. 27) called $2^p—1$ a Lucassian if p is a prime of the form $4k + 3$ and $2p + 1$ is a prime, stating that Lucas had proved that $2^p — 1$ has the factor $2p + 1$. Cunningham considered it probable that primes of the form $2^x \pm 1$, $2^x \pm 3$, (if not yielding Lucassians) generally yield prime values of $2^p — 1$ and that no other primes will. He asserted that all known and conjectured primes $2^p — 1$, with p prime, fall under this rule.

We shall next consider an interesting chain theorem (for its history see Dickson, l. c., pp. 48-50) which can be most conveniently stated by means of a notation defined as follows: Let $s(n) = s^1(n)$ denote the sum of the divisors of n which are less than n; and write $s^k(n)$ for $s[s^{k-1}(n)]$. E. Catalan conjectured that for a given n the function $s^k(n)$ of k has a limit λ, where λ is unity or a perfect number.† J. Perrott (Perott) noticed that the value oscillates if n is one of a pair of amicable‡ numbers, 220 or 284, for instance. A. Cunningham stated that for most numbers n we have $s^k(n) = 1$ for a suitable value of k. There are some numbers n (even some smaller than 1,000) for which

* A number **a** is said to be a **primitive root** of a prime **p**, if a^{p-1} is the lowest power of **a** which is congruent to unity, modulo **p**.

† A number **n** is said to be **perfect** if $s(n) = n$.

‡ Two numbers **m, n** are said to be **amicable**, if $s(n) = m$ and $s(m) = n$.

$s^k(n)$ increases beyond the practical power of calculation. One such number is $n = 138$. L. E. Dickson called the chain n, $s(n)$, $s^2(n)$, . . . periodic of period k if $s^k(n) = n$. If a chain has a periodic component it may be called periodic. Dickson stated the empirical theorem of Catalan in the corrected form that every non-periodic chain contains a prime and verified it for a wide range of values of n. For a perfect number the chain is of period unity. For one of a pair of amicable numbers the chain is of period 2. If n is less than 6233 there is no chain of period 3, 4, 5, or 6. P. Poulet discovered the chain of period 5:

$$n = 12496 = 2^4 \cdot 11 \cdot 71,\ s(n) = 2^4 \cdot 19 \cdot 47,\ s^2(n) = 2^4 \cdot 967,\ s^3(n) = 2 \cdot 23 \cdot 79,\ s^4(n) = 2^3 \cdot 1783,\ s^5(n) = 2^4 \cdot 11 \cdot 71 = n.$$

He also noted that 14316 leads a chain of 28 terms. I am not aware of the known existence of other periodic chains of period greater than 2; it seems probable that no such chains have been noted.

It would be of some interest to have further experimental evidence concerning the character of these chains. In particular, it would be interesting to know periodic chains of periods of 3 and 4. It is also desirable to have a similar experimental examination of a second type of chain described in the next paragraph.

Let $\sigma(n)$ denote the quotient of the sum of all the divisors of n by the least prime factor of this sum and write $\sigma^1(n) = \sigma(n)$, $\sigma^k(n) = \sigma[\sigma^{k-1}(n)]$. Then consider the sequence n, $\sigma(n)$, $\sigma^2(n)$, $\sigma^3(n)$, . . . If n is an even perfect number, it is easy to show that $\sigma(n) = n$. We have $\sigma(48) = 62$, $\sigma(62) = 48$; thus we have a periodic chain of period 2. We have also $\sigma(2^6\ 7) = 2^2 \cdot 127$, $\sigma(2^2 \cdot 127) = 2^6 \cdot 7$, affording a second periodic chain of period 2. It may be shown generally that if $2^p - 1$ and $2^q - 1$ are distinct primes, then the two numbers $2^{p-1}(2^q - 1)$ and $(2^p - 1)2^{q-1}$ form a periodic chain of period 2. We have $\sigma(2^3 \cdot 3) = 2 \cdot 3 \cdot 5$, $\sigma(2 \cdot 3 \cdot 5) = 2^2 \cdot 3^2$, $\sigma(2^2 \cdot 3^2) = 13$, $\sigma(13) = 7$, $\sigma(7) = 2^2$, $\sigma(2^2) = 1$, $\sigma(1) = 1$, . . . Hence some chains may end by repeating unity indefinitely. Again $\sigma(11) = 6$, $\sigma(6) = 6$, so that a chain may end by repeating a perfect num-

ber indefinitely. Again, $\sigma(7\cdot 73) = 2^3\cdot 37$, $\sigma(2^3\cdot 37) = 3\cdot 5\cdot 19$, $\sigma(3\cdot 5\cdot 19) = 2^4\cdot 3\cdot 5$, $\sigma(2^4\cdot 3\cdot 5) = 2^2\cdot 3\cdot 31$, $\sigma(2^2\cdot 3\cdot 31) = 2^6\cdot 7$, $\sigma(2^6\cdot 7) = 2^2\cdot 127$, $\sigma(2^2\cdot 127) = 2^6\cdot 7$; hence this chain finally repeats two numbers indefinitely. Any chain which thus comes to repeat one or more numbers indefinitely may be called a periodic chain. Is it true that all of the chains of this paragraph are periodic in this sense? This seems to be a hard question to answer.

TEACHING PUPILS HOW TO STUDY MATHEMATICS

By ALFRED DAVIS
Soldan High School, St. Louis, Mo.

I

Of all the considerations connected with the study of mathematics, and indeed with the study of any subject, the most important is the mastery of the art of study itself. No topic has been so generally overlooked and neglected heretofore. This neglect is the source of many of our difficulties in teaching, and of many of the criticisms that have been heaped upon the various studies. A supervisor of the high schools of one of our states recently made a general outline for the reorganization of the courses in mathematics for the state. The matter of teaching pupils how to study had been entirely overlooked. His attention was called to the matter, and, realizing its importance, he made it a part of the program. After all, this is the chief thing to be gained from our schools. The pupil must learn the "how." The "what" is not of so great importance. The "what" frequently changes. The "how" is relatively constant. In other words, the pupil should learn how to attack a problem with economy of time and effort, and with the greatest efficiency. The information he gains in the process is incidental, and illustrative of what he ought to expect as a result of his efforts after he has been trained. It is this sort of training that gives the educated man a measure of his powers, and ability to use these powers in the most effective manner in the various problems which he meets in his daily living. If education fails in this it fails utterly; indeed, it is then not education at all, it is a farce, and the school is a failure. Yet this is the point at which the schools do fail most lamentably. No subject in the high school curriculum is equal to mathematics in its opportunities for teaching the art of study. Geometry is especially valuable for this purpose.

If the pupil is to realize the values to be obtained from the study of mathematics, it will usually be necessary that he master more completely the art of study, than he has done in the pur-

suit of his other studies. Indeed the ability to master mathematics means the mastery of this art, while failure in mathematics means inability to study properly. Heretofore it has been assumed that anyone who is willing to work hard can master the technique of study—that if teacher and parents, by combining their efforts, and with coaxing, urging, or threatening, can secure application and effort on the part of the pupil, all other desirable results must, in some unexplained and unexplainable way, follow. Nothing could be more seriously in error. The pupil with his book before him reads, reads, reads; and then, often with a frantic and usually futile effort, he tries to remember what he has read. He usually fails in this, but if he should succeed with the memory work and give what is considered a good recitation, there is little or no real gain to the pupil. In case of failure the teacher usually makes additional suggestions, such as reading aloud, or writing to aid the memory. Little, if any, reference is made to the fact that *thinking* is real study, and that one may read and recite indefinitely without doing any studying. Indeed, the things of secondary importance—learning and reciting facts—are emphasized so much, as to exclude the possibility of real study. While we assume that the pupil needs training for his vocation, this most important of all arts is left for him to learn without assistance. This is unfair to the pupil since it robs him of his rightful heritage—the right to benefit from the experience of others as far as is possible. Is it any wonder that so many pupils fail in school; or, at best, make an indifferent success of their work? Is it any wonder that mathematics, in which it is so absolutely necessary that one *study* to make real progress, or to get real enjoyment from it, is a failure, and that so much adverse criticism is heaped upon it? Teaching pupils how to study is the first and most important duty of the teacher.

One of the most serious difficulties lies in the fact that teachers themselves do not know, and hence, cannot teach pupils how to study. Dr. Lida B. Earhart, of Columbia University (in *"Teaching Children to Study"*), in connection with an experiment in graded schools to ascertain whether children were taught how to study, says:

"Teachers themselves are lacking the proper conception of the process of higher study; they tend to exalt memorizing; and they do not, as a class, accord recognition to any factor or factors as being essential to study. In several instances, the factors which they have recognized were employed largely by the pupils in their studies; and the factors which the teachers have overlooked in their resports were used but little by the pupils in their tests. . . . Although pupils possess ability to employ the various factors of proper study, the teachers lack a clear conception of what such study is. The teachers who wrote the questionnaires do not themselves employ these factors to any great extent; and the teachers observed in the classrooms are not training their pupils to use them. The teacher is the center and the moving power in nearly all of the work, and the requirements laid upon the pupils involve mechanical effort to a large degree. The aim of the work as a whole seems to be the mechanical acquisition of subject-matter.

"The development of the power to work independently, in telligently, and economically is almost entirely ignored. The teachers do not know of what such study consists and consequently give little thought to its cultivation. They would probably do so if they had definite ideas as to its nature, for they are frequently heard to lament the fact that their pupils do not know how to study or to think. Unfortunately the books on method give little or nothing in regard to the method of study. They deal almost exclusively with the teaching side of the school room situation, and do not say anything at all about training pupils to study, or else what they do contain is stated in such general terms that it benefits the teacher very little."

The teaching of pupils how to study should be made a necessary and an important part of the preparation of every teacher. However, the time to begin to teach the art of study, for both teachers and pupils, is at the beginning of their work in the grades.

Can pupils be taught to study? Do the difficulties connected with its mastery put it beyond the reach of elementary, and even of high school pupils? Dr. Earhart, on the basis of her experiment with fourth grade pupils, concludes that:

"Pupils in the elementary schools in grades including the fourth, as well as higher classes, are able, not only to employ the factors of logical study, but also by means of systematic effort they can be made to improve in their employment of them."

The pupil may or may not be conscious of the fact that he is learning how to study. That is not an important matter in the lower grades. However, it is more than likely that the interest of the pupil in his work would be increased by knowing the aim of the teacher, provided he has been shown the value of having the ability to study—a power enabling him to solve problems out of, as well as within, the classroom. While the pupil may not know the full purpose in what he does, the teacher must be very clear in the matter. She must also be familiar with the way, so as to direct the efforts of the pupil effectively towards the mastery of study. The pupil will, later, of course, have a full knowledge of the importance of what he has done, even if he does not know now. If the pupil is not given the proper aid by the teacher, he may succeed by accident, but he will rarely succeed so well that he might not have done better with proper help. If the teacher is not skillful in giving assistance there may be great waste of the pupil's time even though he is fairly successful; but, more serious than this there is the likelihood of developing improper and wasteful habits of study which will be remedied with great difficulty, or probably not at all, and so discount the pupil's powers and possibilities for his entire life.

Professor F. M. McMurry, in *"How to Study,"* says: "Bad methods of study easily become a serious factor in adult life, acting as a great barrier to one's growth and general usefulness."

A class with a teacher who does not know how to study is only a caricature of what a class ought to be. Whatever else a scheme for education may have, if it has teachers who lack this knowledge, there is no such thing as education, in a real sense, possible. Failure to teach pupils how to study is a menace to our national welfare, it is a crime against humanity.

Those matters which affect the health and physical conditions of pupils are of very great importance and should receive attention first. It frequently happens that class sluggishness is

due to foul air. Indeed, pupils need instruction in the importance of fresh air so that they may properly attend to the matter at home. The lighting should be carefully arranged so as to avoid dim lights, cross lights, glaring lights, etc. The class and study rooms should be made neat and attractive. Slovenly and uncouth surroundings will eventually result in corresponding mental habits. Untidy environment is likely to be morally harmful, leading to lower standards of living. The pupil should be aided in providing himself with such associations as will cultivate a cheerful, hopeful state of mind.

The teacher should be quick to observe physical defects in his pupils. The writer has a number of cases in mind in which the retardation of pupils was entirely due to defects in hearing or in vision, yet the pupils had been in school a number of years before these defects were observed. By proper attention such defects may be removed or materially helped by rearranging the seating of a class, etc.

The pupil must assume, not only the mental, but also the physical attitude of attention. It will aid in securing the proper mental effort to place the body in that condition in which the pupil finds it when he gets the best results from his study. If he loafs the mind will tend to assume a similar condition. All study must be under the power of concentration, with intense application, when the mental energy is at white heat, for it is work under such conditions that gives results of real value. It is under such conditions that new and worth-while ideas will, and have, come to men—no one knows just how nor whence. The world's greatest discoveries have been the result of intense concentration on the problem, living with it, going to the limit of human power, until the light suddenly, but mysteriously broke.

Professor Carmichael says: "From the ore of thought important truth can usually be extracted only under the heat of a glowing zeal, when the mind is surcharged with that determination which arised from strongly motivated activity. Work which proceeds not at white heat is coldly done and possesses not the fire of vitality. . . . One cannot successfully woo the science of mathematics except under the inspiration of high motives. She refuses to consort with sordid aims. She can be happy only with him of high ideals who cherishes her nobler qualities; and

only to him will she yield her increase for the blessing of mankind."

The writer had a pupil in his geometry class recently, who had been weak in her work. Suddenly she surprised the class by giving a perfect recitation on a difficult piece of advanced work. In telling the class how she accomplished this result, she said that at first the problem seemed impossible to her, but that she said to herself that she could and must get it. With this determination she set to work and did not give up until she was sure of complete mastery. This sort of determination is necessary for the success of the student. However, he must not work under such pressure for long periods without change. Forty minutes of such work followed by twenty minutes of relaxation will give better results over a long period than uninterrupted plodding. The student should work without worry, fluster, or nervousness, and he should be constantly on the alert, as he studies, for improvements in his methods of work. This sort of work is impossible if one goes immediately to studying after eating. Then, too, it is not possible to go immediately from intense work in one subject to the same sort of work in another unrelated subject. To be convinced of this one has but to try the experiment. One might as well expect a train under high speed to reverse itself instantly. The machinery used by the mind seems to be subject to the same laws, as regards momentum, as prevail in the physical world. The mind must have time to readjust itself before it can follow effectively a new line of effort. This fact needs more attention in our teaching. It is frequently overlooked. We often expect the pupil to change immediately from mathematics to history, from history to physics, from physics to language, etc.; the result is, that the student gets the wrong notion of what study is. We defeat ourselves in trying to educate him according to this plan.

The student must have faith in his ability to do the work assigned, unless he has this he cannot apply himself in such a way as to get results; without this faith the battle is lost before it is begun, he does not expect success and is not likely to get it. The pupil ought to remember that the work assigned is within his power to do or it would not be required of him. If the work is impossible for the student to do the fault lies with the teacher.

The pupil must have enough information about the assigned work to convince him of its importance and to arouse his interest in it. The advanced work should be studied as soon as possible after the class in that subject, while the interest is strong, and while the facts are fresh in the mind. Before making a detailed study of the new work the student should make a general survey of the field to be covered. This should be followed by a review of the preceding work which may have a bearing on the new problem. If there seems to be a lack of interest he should try to ascertain the cause and proceed to remedy the defect. Interest may be aroused by recalling the values of the topic as far as known, by the desire not to be outclassed by other pupils, or by the satisfaction which one may anticipate in mastering a difficult problem, giving assurance of increased strength, and of ability to meet other difficult situations in life with a corresponding determination to succeed.

The student should pause at frequent intervals during his study of a lesson, for the purpose of recalling points of importance, for comparing these and relating them to other information. It is advisable to have a note book in which should be written a digest, or summary of the lesson in the pupil's own words, rather than in those of the book. The book should be freely underscored and marginal notes written if these seem of value for a rapid review or for comparison. Supplementary leaves may be pasted in the book at the proper place if there is use for them. An unabridged dictionary, encyclopedia, other texts, and all available information should be convenient for reference. These helps should be used when needed for reference. Nothing should be guessed at, a meaning of a word, a statement needing verification, a reference which is not fully understood, these should be looked up immediately. This is a part of real study, any different procedure would develop bad study habits. After the lesson has been studied carefully, the student should select the *most* important points of the lesson and put extra time in the further fixing of these in his mind; about these as centers he should relate the points of less importance. Nor is this enough, he should find, if possible, instances in his own experience where the lesson applies. Finally, the important points, definitions, choice statements should be

memorized. But at this juncture memorizing is a simple matter. The important parts will usually fix themselves in the memory because they have been thoroughly mastered. This procedure followed by careful attention to the recitations and discussions of the class when the lesson is considered there, will almost guarantee success to the student. The process may seem laborious at first but when it becomes a habit it will take less time, and the pleasure and profit will give returns a thousand fold. Such a student will have developed self-reliance. He will appreciate the spirit of such men as Grant, who would not allow anyone to help him solve a problem in his mathematics.

If the lesson should be mathematics the pupil should form the habit of checking his work. This will not necessarily shake his confidence in himself, but it should lead him to habitually suspend judgment on an issue until he has weighed all available evidence in the case, and until he is reasonably sure of his conclusions. This is the sort of person whose judgment on important matters is eagerly sought by his fellows—he becomes one of the influential persons in his community.

If the pupil has failed with his lesson after doing the best he knows how, it is a good plan, occasionally, to have him tell the class his method of study. This may be handled in such a way by the teacher that he will be able to correct his methods of work, and others will get valuable suggestions; to say nothing of the valuable hints the teacher himself will get regarding the individuals of the class. However, the spirit of helpfulness must be maintained, in other words, the discussion must be constructive, and not add to the discouragement of the pupil. Valuable suggestions may also be had by having a successful pupil tell a class of his method of work, providing he can do it in the proper spirit.

Each pupil must be trained to use the teacher to the utmost. He should feel personal responsibility in seeking out the teacher to clear up obscure points in the lesson, especially lessons that have already been studied in class. Of course it should not be too easy to get help in this way; the pupil must learn to do his utmost first. Work with individuals should never replace the work which would be more valuable when given in class. Practically all teaching, whether it is the subject or how to study

the subject, should be made a class affair. Teaching individuals is a waste of time and effort. This experiment was tried recently in the University High School of one of our State universities. After teaching a class in mathematics individually for two years, the teacher declared it a failure.

The preparation for examinations usually gives unnecessary worry to the student. A little instruction as to the purpose and value of examinations would aid greatly. Their chief value is not to determine what grade the pupil ought to get for his work in the subject; although, in case of doubt, they may be valuable in this respect. The teacher is usually sufficiently well informed as to the rank of the pupil before the examination is given. The examination has its chief value in the preparation the pupil makes, providing this is done in the proper manner. The examination gives opportunity to review the work covered during an extended period. The important points are selected and related to each other. In a sense, a bird's eye view is taken over a considerable portion of the work so that it can be seen in its larger and more important relations to other problems and to life in general. Just as the subject should begin with a general survey of the field to be covered, for the sake of unity, so the course should end with a review for the same purpose. This will impress upon the mind a consciousness of the distinct gain and achievements in the course—the real work that has been done and its value. In preparing for the examination the student should go over the entire field picking out the important points, and making a list of the points which he recalls with greatest difficulty. The next effort should be made upon this new list; after study attempt to recall the points; make a new list of the points which still give difficulty. Continue this process until all the points selected as of first importance are thoroughly well known. This process may be repeated, if necessary, and if there is time, for the points of secondary importance. However, if the examination is a fair one, that is if it does not contain unimportant topics and purely catch questions, the student should be well prepared after he has mastered the most important topics. His power of discrimination will have been developed while studying the course, providing both he and the teacher have been doing conscious work. The chief source of

worry for the student results from neglect of his work during the term. There is no excuse for this and there is really no remedy. Neglect of work during the term defeats the whole purpose of the study whatever grade may be obtained. It is, unfortunately, possible for pupils so disposed to "bluff," and scheme their way through. This sort of thing is likely to result in disappointments later in life.

(*To be continued*)

LA DISME OF SIMON STEVIN—THE FIRST BOOK ON DECIMALS

By VERA SANFORD
The Lincoln School

The history of the decimal fraction has two distinct sections: the evolution of the idea of decimals, and the evolution of a convenient symbolism. The idea might, indeed, be traced to the very beginning of the place-value of the Arabic notation. The symbolism has not been standardized today, as witness the English 3·14, the French 3,14 and our 3.14. Yet, strangely enough, the theory of operations with decimals appeared in complete form in the first work devoted wholly to the subject, *La Disme* (1585) by Simon Stevin of Bruges.

It is the purpose of this sketch to show, by excerpts from *La Disme*, what Stevin did with decimals and what uses he anticipated men would make of them. Since his introduction, definitions and symbols give a clue to the way in which he came to invent these numbers, a reconstruction of his line of thought is pertinent to the case, even though this may degenerate into merely what one thinks he would have thought had he been Stevin. And to give *La Disme* its proper setting, there must be added a note on computations with fractions in the sixteenth century.

The devices that might have developed into the decimal fraction fall into two groups: those which depend specifically upon the decimal arrangement of the Arabic notation, and those which involve symbols that are happy accidents or actual forerunners of the decimal point, according to one's interpretation of their use. In the first group was a scheme for finding the nth root of a number. This may be expressed in modern symbols as $\sqrt[n]{a} = \frac{\sqrt[n]{a \cdot 10^{kn}}}{10^k}$. Its value lies in the fact that it enables one to find a root to any required degree of accuracy and that fractions enter only in the last step of the work. In a similar way, interest was computed on a principal 100,000 times too large, and trigonometric functions were found from a circle of radius

10,000,000. Symbols typical of the second group are the following: in dividing by multiples of 10, 100, 1,000, Pellos (1492) used a period to mark off one, two or three places in the dividend as if to indicate that digits to the right belonged inevitably to the remainder; and Rudolff (1530) printed a compound interest table much as we would save, that a bar (/) acts as a decimal point. Several other writers used symbols of this sort, but no one before Stevin explained what they actually signified.

Any one of these devices might have been the beginning of the decimal fraction, but a careful study of *La Disme* indicates that Stevin approached the subject from a different standpoint—that of adapting to everyday use and to the current number system a method devised by scholars of an earlier time. Stevin was well qualified to appreciate the learning of the past and the difficulties of his contemporaries. He lived in the Netherlands in the days of the great struggle against Spain. His ability in mathematics and physics, and his skill in applying his knowledge to military and civil matters, won for him the post of adviser to Maurice of Nassau, the son of William the Silent. Thus Stevin was in close touch with the affairs of a country which necessity had driven to seize every aid that science could give, and he was collaborator with a prince who realized this necessity most keenly.

In all such work, numerical computations must have played a large part, and although methods such as are noted above were in use for special cases, there was no satisfactory general scheme for dealing with fractions.

Common fractions were difficult to use and unsatisfactory in expressing results. The extremes to which they were carried in an effort for accuracy is shown in the case where Stevin gives $233\frac{2356847}{23476796}$ ℔. for the present value of an annuity of 54 ℔. yearly for 6 years, interest at 12%. Stevin was not alone in using large denominators. He makes occasional attempts to reach significant figures in his results, as in computing his interest tables where he counts $\frac{100}{101}$ as 1 "for it is greater than one-half," but in *La Disme* he considers that the 0.00067

A discussion of these devices is given in **The Invention of the Decimal Fraction,** by D. E. Smith, Teachers College Bulletin, First Series No. 5.

in 301.17167 is of no account, and he writes his result as 301.171.

Stevin's decimals seem to have evolved from the sexagesimal fractions of the scientists. Sexagesimals were really a series of denominate numbers in which a unit was 60 minutes (primes), a minute was 60 seconds, a second was 60 thirds and so on.[1] These numbers were inherited from the Babylonians and the Greeks, and they are still used in our divisions of the hour and the angle. The advantages of sexagesimals in expressing results are obvious. Two such numbers could be readily compared. Many fractions could be expressed exactly in sexagesimals of the first order, and others could be given in combinations of minutes, seconds and thirds. In one case, for instance, an approximate root of an equation is carried to the sexagesimal of the tenth order. These numbers had grave disadvantages, however. They were of little use if one wished to extract roots, and the multiplication or division of two sexagesimals was difficult.

It is not surprising that Stevin felt obliged to write a careful introduction to the work that was to revolutionize all computations. One can imagine the scepticism that would greet a pamphlet of 36 octavo pages labeled with that ambitious purpose, and Stevin's attempts to reconcile the size of his book to its sub-title, and his desire to clear himself from implications of undue boasting make his introduction fully as interesting as his theory. As for the book itself—it first appeared in Flemish under the title *La Thiende* (Leyden, 1585). In the same year, a French translation *La Disme* was printed as an appendix to Stevin's *l'Arithmetique.* In the next half century, the work went through at least six editions, one of them a translation into English (1608).[1]

Let us suppose for the moment that we have never heard of decimals and that we are among the people to whom Stevin dedicated *La Disme.* In this case his introduction may sound like a fairy tale that we wish were true, but that we fear is not. We shall be interested, however, to learn what it is that makes the author at once so confident and so modest. Any theory that will

[1] **Pars minuta prima, pars minuta secunda, etc.**

[1] The translation quoted in this article was made from Girard's second edition of **Les Oeuvres Mathematiques de Simon Stevin** (Leyden, 1634).

take such hold of a man is worth considering, so let us begin *La Disme* which Stevin called "La Practique des Practiques"—we might call it the "super-method."

LA DISME

Teaching how all Computations that are met in Business may be performed by Integers alone without the aid of Fractions.

Written first in Flemish and now done into French
by
Simon Stevin of Bruges

To Astrologers, Surveyors, Measurers of Tapestry, Gaugers[2], Stereometers[3] in general, Mint-Masters, and to all Merchants.
Simon Stevin sends Greeting.

Any one who contrasts the small size of this book with your greatness, my most honorable sirs to whom it is dedicated, may think my idea absurd, especially if he imagines that the size of this volume bears the same ratio to human ignorance that its usefulness has to men of your outstanding ability; but, in so doing, he will have compared the extreme terms of the proportion which may not be done. Let him rather compare the third term with the fourth.

But what is it that is here propounded? Some wonderful invention? Scarcely that, but a thing so simple that it hardly deserves the name invention; for it is as if some stupid country lout chanced upon great treasure without using any skill in the finding. But, if anyone thinks that, in explaining the usefulness of **La Disme,** I am boasting of my cleverness in devising it, he shows without doubt that he has neither the judgment nor the intelligence to distinguish simple things from difficult, or else that he is jealous of the common good. However this may be, I shall not fail to mention the usefulness of this thing even in the face of this man's empty calumny. But since the mariner who has found by chance an unknown isle, may declare all its riches to the king as, for instance, its having beautiful fruits, pleasant plains, precious minerals, etc., without its being reputed to him as conceit; so may I speak freely of the great usefulness of this invention, a usefulness greater than I think any one of you anticipates, without constantly priding myself on my achievements.

Stevin here enumerates cases which show the usefulness of number in the work of the astronomer, the surveyor, the mintmaster, and the merchant. He stresses the difficulties of manipulations with sexagesimals and with denominate numbers and speaks of the almost inevitable errors in calculation that vitiate excellent work. Stevin claims that *La Disme* teaches how these computations may be performed by whole numbers and he says that work tedious even to a skilled computer may now be accomplished with the same ease as in reckoning with counters.

---

2 Gaugers: men whose business was the measuring of wine-casks, an important thing in connection with the excise duties, and necessitated by the great divergence in the size and shape of the barrels of the time. Gauging lingered in the American arithmetics well into the last century.

3 Stereometers: men who measured volumes of all sorts. Stevin later is at great pains to show that, although all gauging is stereometry, all stereometry is not gauging.

The force of the comparison is apparent only when one reflects that counter-reckoning in Stevin's time was a device used principally by people little skilled in the finer points of working with Arabic numerals—its greatest value was in addition and subtraction. Then comes Stevin's reason for presenting this to the public:

> If by these means, time may be saved which would otherwise be lost, if work may be avoided, as well as disputes, mistakes, lawsuits, and other mischances commonly joined thereto, I willingly submit **La Disme** to your consideration. Someone may raise the point that many inventions which seem good at first sight are of no effect when one wishes to use them, and as often happens, new methods, good in a few minor cases, are worthless in more important ones. No such doubt exists in this instance, for we have shown this method to expert surveyors in Holland and they have abandoned the devices which they had invented to lighten the work of their computations and now use this one to their great satisfaction. The same satisfaction will come to each of you, my most honorable sirs, who will do as they have done.

The compactness of the book implies no lack of formality. The *Argument* outlines the topics to be treated under *Definitions* and *Operations* and promises that "At the end of this discussion there will be added an *Appendix* setting forth the use of *La Disme* in real problems." How modern it all is! Motivation that would whet anyone's interest, a thorough preliminary testing out which we are assured was successful, and, as a climax, real problems!

We come then to the actual theory.

Definition I

La Disme is a kind of arithmetic based on the idea of the progression by tens, making use of the ordinary Arabic numerals, in which any number may be written and by which all computations that are met in business may be performed by integers alone without the aid of fractions.

Explanation

Let the number one thousand one hundred eleven be written in Arabic numerals as 1111, in which form it appears that each 1 is the tenth part of the next higher figure. Similarly, in the number 2378, each unit of the 8 is the tenth part of each unit of the 7, and so for all the others. But since it is convenient that the things which we study have names, and since this type of computation is based solely upon the idea of the progression by tens or disme,[1] as is seen in our later discussion, we may properly speak of this treatise as **La Disme,** and we shall see that by it we may perform all the computations we meet in business without the aid of fractions.

---

[1] Disme: "tithe," later the word was contracted into dime, so our coin has this connection with the first decimals.

Definition II

A whole number is called the **Commencement** and has the symbol ⓪.

Definition III

The tenth part of a unit of the commencement is called a **Prime** and has the symbol ①, and the tenth part of a unit of prime is called a **Second** and has the symbol ②. Similarly for each tenth part of the unit of the next higher figure.

Explanation

Thus 3① 7② 3③ 9④ is 3 primes 7 seconds, 3 thirds, 9 fourths, and so we might continue indefinitely. It is evident from the definition that the latter numbers are 3/10, 7/100, 3/1000, 9/10000, and that this number is 3739/10000. Likewise 8⓪ 9① 3② 7③ has the value 8 9/10, 3/100, 7/1000, or 8 937/1000. And so for other numbers. We must also realize that in **La Disme** we use no fractions and that the number under each symbol, except the commencement, never exceeds the 9. For instance, we do not write 7① 12②, but 8① 2② instead, for this has the same value.

Stevin's choice of names and symbols for these numbers is explained in the first pages of *l'Arithmetique*, which was published in the same volume as *La Disme*. Here, he calls the terms of a geometric progression by the ordinals: prime, second, third, . . . with the signs ①, ②, ③ . . . giving as examples ①2 ②4 ③8, ①3 ②9 ③27. In *l'Astronomie*, he uses these symbols in writing both decimals and sexagesimals. In *l'Arithmetique*, he extends this to the case where the ratio is an unknown quantity. Thus ①, ②, ③ stand for x ,x^2, x^3. The commencement, as defined in *l'Arithmetique*, was an integer or an irrational number used in algebraic computation. The symbol ⓪ was to enter only when the commencement was an abstract number. Denominate numbers in Stevin's other works and in the Appendix to *La Disme*, appear as 1 hour 3① 5②, 5 degrees 4① 18②, 2790 verges 5① 9②. The origin of the symbol for the commencement has no connection with the zero exponent. It is probably a direct consequence of the fact that the mediaeval writers left a blank space between the units and the first place of sexagesimals. These writers had denoted the orders of the fractions by suitable abbreviations or sometimes by numerals, either written above or designated by some distinguishing mark. The symbol ° for a degree first appeared in print in 1586. In *l'Arithmetique*, Stevin notes that Bombelli uses the same symbols with the exception of the ⓪. [These quantities were written by Bombelli (1572) as ◡1, ◡2, ◡3.] One concludes that Stevin borrowed his symbolism directly from Bombelli, but as the latter

writer had felt no need for a symbol corresponding to the ⓪, Stevin was forced to invent one for himself, and the zero was the logical outcome, as it fitted with the rules which he laid down for the multiplication and division of the numbers of *La Disme*. Yet he never seems to have connected these symbols with the exponents of the tens in the donominators they replaced!

In 1525, Finaeus had unwittingly provided a precedent for Stevin's use of the same symbols for decimals and sexagesimals, by writing the product of 8 by 42 (minutes?) as 5.36, *i. e.*, $5\frac{3}{6}\frac{6}{0}$. Subsequently, Beyer (1616) used both the comma and the ′, ″ notation for decimals. Napier used ′, ″, etc., in his *Rabdologia* (1617) but in his *Constructio* he shifts to the comma which Burgi had used in 1592. Dr. Glaisher[1] gives the opinion that Stevin used his array of symbols to make the numbers of *La Disme* more like those to which his contemporaries were accustomed. But one point should be mentioned here. In *La Geometrie*, expressions such as these occur: 1875③ $= \frac{3}{4} \times \frac{9}{4} \times \frac{5}{8}$ and $7 \times \frac{2}{5} =$ 280②. It is possible that this omission of the signs was made by the editor Girard; but by 1625, the date of the publication of his first edition of Stevin's works, Girard would have had ample opportunity to see the advantage of the comma separatrix which Burgi was using. Had he been making changes in Stevin's symbolism, these influences would have led him to retain the ⓪ and drop the ③ or the ②. Similarly, if Stevin was influenced by the bar of Rudolff etc., we would have expected him to do likewise. But Stevin appears to have thought of his 1875 as 1875 thirds rather than as 1.875.

Stevin's explanation of Definition I and his choice of names bear out our hypothesis that his work with decimals resulted from an effort to devise a system of numbers analogous to sexagesimals in which the place-value of the Arabic notation would function. His retention of the symbol of the last number, rather than that of the commencement, seems to indicate that he was not guided by the sporadic inventions of Rudolff and the rest. Yet, at first sight, these appear to be the more probable group of ancestors for the decimal fraction!

---

[1] Logarithms and Computation, Napier Tercentenary Volume p. 77.

Operations with the numbers of *La Disme* are treated in four propositions: Addition, Subtraction, Multiplication and Division. Each contains a formal statement of what is given and of what is to be proved. Careful reference is made to the corresponding work with integers in *l'Arithmetique*. The rigid uniformity of these propositions permits us to judge their style from a single example, and their content may be summarized without including the rather tedious proofs which are identical in method with the proof in Proposition I.

Proposition I. To add numbers of **La Disme.**

Given three numbers of **La Disme,** 27⓪ 8① 4② 7③, 37⓪ 6① 7② 5③, 875⓪ 7① 8② 2③.

Required: To find their sum.

Construction: Arrange the numbers in order as in the accompanying figure, adding them in the usual manner of adding integers.

			⓪	①	②	③
		2	7	8	4	7
		3	7	6	7	5
	8	7	5	7	8	2
	9	4	1	3	0	4

This (by the first problem of **l'Arithmetique**) gives the sum 941304, which, as the symbols above the numbers show, is 941⓪ 3① 0② 4③. And this is the sum required.

Proof: By the third definition of this book, the given number 27⓪ 8① 4② 7③ **is 27 8/10, 4/100, 7/1000 or 27** 847/1000. Similarly, the 37⓪ 6① 7② 5③ **is 37 675/1000 and the** 875⓪ 7① 8② 2③ **is 875** 782/1000. These three numbers **27** 847/1000, **37** 675/1000, **875** 782/1000, added according to the tenth problem of **l'Arithmetique** give **941** 304/1000, but 941⓪ 3① 0② 4③ has this same value, and is, therefore, the true sum which was to be shown.

Conclusion: Having been given numbers of **La Disme** to add, we have found their sum, which was to be done.

Note: If, in the numbers in question, some figure of the natural order be lacking, fill its place with a zero. For example, in the numbers 8⓪ 5① 6② and 5⓪ 7② where the second number lacks a figure of order prime, insert 0①, and take 5⓪ 0① 7② as the given number and add as before.

	⓪	①	②
	8	5	6
	5	0	7
1	3	6	3

Stevin has three ways of writing these numbers, the symbols may be placed above or below the numbers or after the digit which they qualify. It should be noticed that Stevin shifts decimals to common fractions, never vice versa. And although he uses a zero to fill a gap in his decimals, he feels no obligation to

begin with the prime. In Proposition IV, for instance, he writes 3④ 7⑤ 8⑥.

In multiplication, the symbol of the last number of the product is found by adding the symbols of the last terms of multiplicand and multiplier. Stevin gives as proofs the case of 2① multiplied by 3②. The product, according to the rule, should be 6③. The values of the given numbers are $\frac{2}{10}$ and $\frac{3}{100}$ and the value of the supposed product is $\frac{6}{1000}$, but this is identical with the product of 2① by 3②, which was to be shown.

In division, the symbol of the last term of the quotient is found by subtracting the symbol of the last term of the divisor from that of the dividend. If the numbers of the divisor should be of higher order than those of the dividend, zeros must be added to the latter.

It sometimes happens that the quotient cannot be expressed by whole numbers, as in the case of 4③ divided by 3②. Here, it appears that the quotient will be infinitely many threes with always one-third in addition. In such a case, we may approach as near to the real quotient as the problem requires and omit the remainder. It is true indeed that 13⓪ 3① 3⅓②, or 13⓪ 3① 3② 3⅓③ is the exact result, but in **La Disme**, we propose to use whole numbers only, and, moreover, we notice that in business one does not take account of the thousandth part of an ounce or of a grain. Omissions such as these are made by the principal geometers and arithmeticians even in computations of great consequence. Ptolemy and Jehan de Montroyal, for instance, did not make up their tables[1] with the utmost accuracy that could be reached with mixed numbers, for, in view of the purpose of these tables, approximation is more useful than perfection

Finally, square roots may be extracted by first making the last symbol an even number by the addition of a zero if necessary, then getting the root as with integers. The symbol of the last number of the root will be one-half that of the given number. "Similiarly, for all other roots."

The *Appendix* proposes to show how all computations that arise in business may be performed by the numbers of *La Disme*. Its greatest interest to us is the treatment of units of area and volume, and the extension of the numbers of *La Disme* to less obvious applications. We may safely guess that the sections of

[1] The tables of Ptolemy to which Stevin had access were probably those in the translation of the Almagest made by Peurbach and Regiomontanus (Stevin's Montroyal) in the 15th century. In these tables trigonometric functions were computed from a circle of radius 10,000,000.

the least interest to the readers of the sixteenth century are of the greatest interest to us today, and conversely.

Article One
Of the Computations of Surveying

When the numbers of **La Disme** are used in surveying, the verge[2] is called the commencement, and it is divided into ten equal parts or primes, each prime is divided into seconds, and, if smaller units are required, the seconds into thirds, and so on so far as may be necessary. For the purposes of surveying, the divisions into seconds are sufficiently small, but in matters that require greater accuracy as in the measuring of lead roofs, one may need to use thirds. Many surveyors, however, do not use the verge, but a chain of three, four or five verges, and a cross-staff* with its shaft marked in five or six pieds with their doigts. These men may follow the same practice here, substituting five or six primes with their seconds. They should use these marks without regard to the number of pieds and doigts that the verge contains in that locality, and add, subtract, multiply and divide the resulting numbers as in the preceding examples. . . . To find the number of pieds and doigts in 5 primes 9 seconds (the result in one of the examples cited) look on the other side of the verge to see how many pieds and doigts match with them; but this is a thing which the surveyor must do but once, i. e., at the end of the account which he gives to the proprietaries and often not then, as the majority of them think it useless to mention the smaller units.

This last illustrates a consequence of the tremendous divergence in units of measure resulting from the feudal conditions of Mediaeval Europe, where each petty ruler ordained such measures as he saw fit—standards that might or might not bear recognizable relation to other units of the same name in other parts of the same country.

Stevin's examples of the use of the numbers of *La Disme* in surveying consist of adding and subtracting areas, finding the area of a rectangle, and determining where a line should be drawn to cut a rectangle of given area from a given rectangle. In all of this work, the prime of a unit of square measure is one-tenth of the unit itself.

Article Two advises that the aune of the measurer of tapestry be divided into tenths as was done with the verge of the surveyor.

2 The verge was a unit of both linear and square measure, and the word was also used for the surveyor's rod.

* The cross-staff is a piece of wood mounted at its midpoint perpendicular to a shaft and free to move along this shaft to other positions parallel to the first one. To use this instrument, the observer adjusts it so that the lines of sight from the end of the staff to the tip of the cross-piece coincide with the endpoints of the lines to be measured. Distances are computed from one measurement and similar triangles. When neither end of the required line is accessible, the distance is computed from two observations at known distances from each other. The pied and doigt were units approximately equal to our foot and inch.

Article Three deals with the measurement of casks. Stevin says that he has made his demonstration brief because he is writing to master gaugers, not to apprentices. As a result it is almost unintelligible. He gives great emphasis to the division of the measuring rod by points spaced according to the square roots of 0.1, 0.2, 0.3, . . . with intermediate points corresponding to the square roots of 0.11, 0.12, . . . This division is made by finding the mean proportional between the unit and its half which gives a point answering to $\sqrt{0.5}$, and the mean proportional between this segment and its fifth part gives $\sqrt{0.1}$. The method of using this is not explained, but it is evident that the diameters measured by this rod would have the same ratio as the areas of their respective sections when measured by the decimally divided rod.

Article Four discusses the numbers of *La Disme* in volume measurement. The illustrative problem is to find the volume of a rectangular column of dimensions 3① 2②, 2① 4②, 2⓪ 3① 5②. The volume is 1① 8② 0③ 4④ 8⑤ 0⑥.

Note: Someone who is ignorant of the fundamentals of stereometry, for it is such a man that we are addressing now, may wonder why the volume of the above column is but 1①, etc., for it contains more than 180 cubes of sides 1①. He should realize that the cubic verge is not 10 but 1000 cubes of side 1①. Similarly, the prime of the volume unit is 100 cubes of side 1①. If, however, the question had been how many cubes of side 1① are in the above column, the result would have been altered to conform to this requirement, bearing in mind that each prime of volume units is 100 cubic primes, and that each second is 10 cubic primes.

Thus, Stevin makes his units proceed strictly in accordance with the decimal scheme. In this way he forestalls the objection to the metric system that its linear, square, and volume units follow the progressions by tens, hundreds, and thousands respectively, but he sacrifices the simple connection between the submultiples of the units of the three systems. The prime and second of his units of area are squares whose sides are $\sqrt{0.1}$ and $\sqrt{0.01}$; the prime and second of his units of volume are cubes whose edges are $\sqrt[3]{0.1}$ and $\sqrt[3]{0.01}$. This is startling, for we think at once of the difficulty of teaching it to an immature person, but it would be more convenient than the metric idea if we, like Stevin, felt a need of labeling each of our subdivisions and wrote 2 cubic decimeters 55 cubic centimeters instead of 2055 cm^3.

Article Five explains that the division of the angle into minutes and seconds was devised so that astronomers might work with integers. The sexagesimal scheme was used "because 60 is commensurate with many whole numbers." Paying all due reverence to the past, Stevin maintains that the decimal progression is even more convenient. He says that he contemplates publishing astronomical tables using the decimal division of the degree, and he ends with a characteristic peroration on the beauties of the Flemish language in which these tables are to be written. As a matter of fact, the tables which Stevin published (1608) were on the old basis. Father Bosmans (*La Thiende*, Louvain, 1920) explains this on two grounds—Stevin did not rejoice in computations as did van Ceulen (which is fortunate for the world, for the impatience which may have cost us a set of tables, gave us *La Disme*); and secondly, large errors would arise in shifting instrument readings to the decimal system for computation, and in then changing the results back again.

Article Six, "On the Computations of Mint-Masters, Merchants, and in general of all States," begins with the thesis that all measures may be divided decimally, and that the largest unit of each denomination should be the commencement. In the case of money, the limit for the divisions would be the first sub-unit that is less than the smallest coin. Instead of half pound, ounce, and half ounce weights, the smaller weights should be the 5, 3, 2, 1 of each order, and the names prime, second, third, etc., should be retained because of the aid which they offer in computation. Stevin illustrates the advantage of the decimal division of money by a problem of exchange—1 marc of gold is worth 36 ℔. 5① 3②, how much is 8 marcs 3① 5② 4③ worth? The value of a simple method of treating such a problem is obvious when we consider that the value of the marc varied in the different cities, and that the number of the smaller units in a marc varied also.

We might give examples of all the common rules of arithmetic that pertain to business as the rules of partnership, interest, exchange, etc., to show how they may be carried out by integers alone and also how they may be performed by the easy manipulation of counters, but as these may be deduced from the preceding, we shall not elaborate them here. We might show by comparison with vexing problems with fractions the great difference in ease between working with ordinary numbers and with the numbers of **La Disme**, but we omit this in the interest of brevity.

Finally, we must speak of one difference between the sixth article and the five preceding articles, namely that any individual may make the divisions set forth in the five articles but this is not the case in the last where the results must be accepted by everyone as good and lawful. In view of the great usefulness of the decimal division, it would be a praiseworthy thing if the people would urge having this put into effect so that, in addition to the common divisions of measures, weights, and money that now exist, the state would declare the decimal division of the large units legitimate, to the end that he who wished might use them. It would further this cause also if all money that is newly coined should be based on this system of primes, seconds, thirds, etc. But if this is not put into operation so soon as we might wish, we have the consolation that it will be of use to posterity, for it is certain that if men of the future are like men of the past, they will not be neglectful of so great an advantage. Secondly, it is not the most discouraging thing to know that men may free themselves from such great labor at any hour they wish. Lastly, though the sixth article may not go into effect for some time, individuals may always use the five preceding articles as it is clear that some are already in operation.

The end of the Appendix.

Stevin's scheme for the decimal division of measures is unlike that of the metric system in three respects:

1. He did not reach the idea of a universal, invarient standard.

2. All of his units proceed according to the progression by tens with the disadvantage noted above.

3. He avoids the multiple units by taking the large unit as the commencement, but, in Article Four, he provides for a change in the unit according to the exigencies of the problem.

These are minor matters, however, in comparison with the idea itself!

The suggestion that individuals may use the decimal divisions of measures brings to mind our use of the decimally divided mile on the speedometer and the milepost, the decimally divided foot of the surveyor's tape, and the grinding of piston heads to the thousandth of an inch. From a scientist of Stevin's type, we would expect a rigorous demonstration of his device; from a person of his prominence in affairs, we would anticipate an application to real problems. But who would have imagined that the first man to write on decimals would have had the vision to see decimal weights and measures, and that, pending the adoption by the state of such a system, he should recommend the very compromises used by individuals today!

NO HOME WORK FOR MATHEMATICS PUPILS

By HORACE C. WRIGHT
The School of Education, University of Chicago

In August, 1919, at a departmental meeting of the University High School mathematics department, I asked for permission to conduct a section of first year pupils without assigning them homework. Permission was granted and the idea was carried out in all the freshmen mathematics classes.

My request was based upon the fact that for several years I had taught a first-year class the first hour after luncheon, 1:05 to 1:55. The class of this period had worked with enthusiasm and success. The pupils felt refreshed and I could generally put over any topic I cared to present. Their attack was so vigorous that I could teach a new topic, give them opportunity for questions and additional explanations, and get them prepared for doing homework successfully.

Then next day the homework came in nearly all perfectly done. But that did not necessarily mean that each pupil had done the work unaided. My pupils came from homes of teachers and frequently from homes provided with tutors. So I decided that the only true check I had on what they could do unaided was from what they did in the class room. For this reason I felt that the outside work had little real value to me as a teacher and could be dispensed with without loss. Then, too, as a parent of two girls in another high school, I knew that the teachers in the subjects other than mathematics would give enough homework to keep the freshmen busy, and, perhaps, the parents, too, if they attempted to supervise the home preparation.

A second consideration that led me to desire the undertaking lay in ten years' experience in teaching a general introduction to mathematics, covering arithmetic, geometry, algebra, and trigonometry, along fusion lines rather than in tandem fashion. I had learned to illustrate one topic by means of another. I could present a new idea in several forms and illustrate it from different fields of mathematics. This had pleased me and the

pleasure seemed to pass over to the pupils. They got the notion that more and more powerful methods of solutions were coming and that eventually we would command means of dealing with almost any kind of a mathematical situation.

In conducting the class I said nothing about not going to give them homework. My plan was to teach a topic, make every effort to present it clearly, and to invite questions about anything not understood. Questions about the topic came only from the better part of the class until after the class has been set to applying the new idea. Then the poorer half discovers that it lacks full knowledge. After sufficient explanation and practice had been given, I had a tryout.

My tryout consisted of selecting a number of type exercises that embodied the principle taught but had increasing difficulties in solution. For instance, in equations with two unknowns, I chose these exercises:

1. $6a - 4b = 2.$
   $5a + 7b = 43.$

2. $\frac{2h}{7} + \frac{5k}{2} = 33$
   $\frac{3h}{4} - \frac{2k}{5} = 17$

3. With two parallels, the interior angles on the same side of a transversal are $(14x - y)^\circ$ and $(6x + y)^\circ$ and their difference is 14°. Find x, y, and all the eight angles.

4. The areas of two triangles having equal bases are 72 sq. in. and 60 sq. in. Twice the altitude of the first, plus 3 times the altitude of the second is equal to 54 in. Find the altitudes.

5. A man gained 8% on one investment and lost 3% on another. If the money invested amounted to $22,000 and the net gain was $440, what was the amount of each investment?

These examples were taken from Breslich's *First Year Mathematics* and the page and number of each was written on the board. Then I sat at my desk and passed judgment on each solution as each pupil completed and handed it to me. No pupil attempted the second problem until he had done the preceding one correctly. In some instances, a pupil spent fifty minutes trying to get the first of a set done correctly, and failed. On the same occasion other pupils would do two, three, four, five, or more correctly. If the day's work for the set was unsatisfactory, the next day was spent in more instruction and practice, and the third day a second test was given.

I continued this method of *teaching* and *testing* with every topic I wanted thoroughly comprehended.

The foregoing description applies to the year 1919-1920. This year, 1920-1921, I am continuing to omit homework entirely from first-year mathematics classes and also almost wholly from second- and third-year classes.

In addition to reducing the time for required work on mathematics to the fifty-minute class room period five times a week, our entire department with 350 pupils and four teachers is working to increase the per cent. of efficiency of our pupils. Our motto is "Teach, Test, and Teach Again." The standard set is 80 to 90 per cent. for every pupil. Those not attaining the standard are taught the topic a second, a third, or a fourth time, or until the principle has been mastered for the time, at least.

A list of excess credit topics has been posted for pupils desiring to do more than the minimum essentials for credit. Besides this class periods have provisions within the hour for the better groups within each class. For instance, while one part of a third-year class takes supplemental instruction and does more assimilative exercises, the stronger part does the parallel part of the Board examinations for several years.

Unless the teacher is a parent, he seldom realizes the hours the pupils struggle over the home assignments each year. Time is thus spent that could be employed in reading, acquiring musical skill, becoming adept in the duties of the home, enjoying the social life of the family, participating in some of the community activities, and best of all, a few additional hours of sleep.

Factors which make for better instruction through the use of this teaching method are: a more careful selection of topics for consideration; a discarding of considerable irrelevant material; an intimate acquaintance with each pupil's difficulties; a complete control of the learning processes; and a more intense working class period.

In the freshman class, which I mention in my first paragraph, only two out of twenty-six failed to pass the first semester of the second-year course, although the class had been distributed and came under different instructors.

THE FUTURE OF SECONDARY INSTRUCTION IN GEOMETRY

By HARRISON E. WEBB
Central High School, Newark, N. J.

If the future of secondary instruction in geometry is to be judged in the light of the past, two thousand years may elapse without any material change. Horace boasted of a monument more enduring than bronze. *Euclid's Elements* has stood like a granite monolith against the intellectual erosion of centuries. This book has seen the decline of great civilizations, the rise and fall of empires, and the birth of great religions. It has passed unscathed through countless successions of languages and dialects. Such is its perfection of detail that all attempts at improvement during its two milleniums of existence have led only to a return to its original form and substance.

And yet the very existence of so perfect a creation of human thought has barred the path of progress in secondary teaching in geometry during the past century. So complete a picture does Euclid present that his teaching has come to be regarded as the embodiment of absolute truth. Only more recent investigations into the nature of symbolic logic have shown that there do exist certain serious lapses from logical accuracy in the Euclidean text, which have been, as a rule, aggravated in many of its more recent imitations. This is an awful blow to philosophical rigorists, but will be welcomed by true lovers of mathematics. A human Euclid is more to be appreciated!

A second result of these investigations will, it is hoped, take the form of a conviction that it is utterly hopeless to attempt to impress the beginner with the refinements of the pure logic of space, or with those of Euclid. The teacher of geometry, to appreciate this fact, must make studies of this character for himself. No teacher should flatter himself that he is competent to instruct pupils in elementary geometry until he has at least made a survey of the field of pure geometry. The reaction of such a study upon elementary instruction will depend largely

upon circumstances. Geometry, more than any subject taught in secondary schools, is "hard boiled." The vicious circle of the college entrance examination, the textbook written to fit the same, the teacher trained to teach from the book, and the normal course designed to enable the teacher to do this, is almost perfect. What changes, whether for better or worse, may be noted in geometry teaching during the past half-century?

One change stands out in the memory of many now living (and teaching) and as well from an examination of a chronological sequence of published texts. It is an increased emphasis upon "originals."

Let it be noted at once that most "originals" are not original at all, as far as the student is concerned. Not a little difficulty in regard to college entrance examinations in geometry lies in the enormous difference, from the examiner's point of view, between a theorem which the candidate has seen before and one which he has not seen. Secondary school teachers realize this difference most deeply, and pay tribute to its significance by combing over innumerable texts for geometric puzzles to present to their classes in the hope of covering all the ground which is likely to afford material for examiners.

It is no disrespect to Euclid or his remarkable book to say that time thus spent is for the most part wasted. The writer is not competent to enter upon a critical discussion of the question of transfer of mental training from one field of thought to another. But the secondary curriculum is overcrowded. If a great many matters of common knowledge are to be covered properly in high school, matters which were unknown to earlier generations, then there is no time to waste upon false intellectual notions. Secondary mathematics in America has in the past eschewed Cartesian analysis, all mention of the conic curves, all reference to the principles of the calculus, and any attention to the real significance of complex numbers. These things were not "required for college." Instead we have had so-called "originals," such as the following:

"If AB and AC are the equal sides of an isosceles triangle ABC, and if on BA the distance BE is taken from B, and on AC produced the distance CD is taken equal to BE, then the line DE is bisected by the base BC."

The stupendous importance of this fact has doubtless escaped the reader's attention, as it has the writer's. If secondary geometry training actually reached the point where this exercise could be demonstrated on a moment's notice, perhaps something could be said for its kind. But usually this is not the case. Instead, the exercise constitutes merely an additional theorem, and its solution is based upon a device not readily thought of.

Exercises such as these have really prolonged the course in plane geometry and have altered the status of the subject from that of a classic to that of a bag of tricks.

If the secondary course in mathematics is to be a real introduction to this branch of learning, the time devoted to this sort of thing had better be spent upon something else. This means that the course in geometry admits of material reduction in time and extent. If the student is given a fair introduction to geometric concepts in the junior high school course, as is most commendably urged by the National Committee, it comes within the bounds of possibility that the course in plane and solid geometry be reduced to one year in time.

This procedure would involve material changes in the character of the subject. In the first place, the distinction between plane and solid geometry, while essential to Euclid's development, is immaterial from a pedagogic viewpoint. Again, certain important geometric relations, such as line-symmetry, which are now all but entirely omitted, open the way for elimination of many of the tedious circumlocutions of the present order of procedure.

Hilbert showed plainly enough the necessity of making Euclid I:4 over into an axiom (or assumption). Not to enter into the details of this question, it would appear equally logical and practical to assume that any two lines, or angles, or polygons, or curved figures, or solid figures which possess line-symmetry are congruent. From this it easily follows that every case of congruence is reducible to symmetric relations, and a large number of geometric theorems follow almost directly.

Symmetry and parallelism are really forms of correspondence. The last word in pure geometry makes the subject a science of correspondence of points. Similarity is another form of cor-

respondence. Similarity in two and in three dimensions is identical. If geometry teachers could be persuaded to strain at a few less gnats and swallow a few more camels, the plane and solid goemetry could be combined into one subject, for which one year's time would be ample. Most of the familiar theorems of solid geometry are no more difficult than much of the plane geometry, and they are no more numerous than the "originals" which now have such a vogue.

Another economy would result from the omission from the plane geometry of a whole chapter of theorems relating to "numerical properties of triangles." These theorems are interesting, but, except possibly for the two which measure the third side of an oblique triangle, they are not essential.

Another section which could be omitted without being missed is that relating to the golden section and the inscription of the regular decagon in the circle. This problem is of interest only in so far as it relates to the question of the superior limits of elementary mathematics in the solution of the pure equation $x^n - 1 = 0$.

In the solid geometry, time would be saved by agreement to accept some form of Cavalieri's theorem as a fundamental assumption.

There is also a question as to whether the course in solid geometry might not be shortened by detaching those theorems which relate to polyhedral angles and spherical polygons, including triangles. Spherical geometry really signifies very little unless it is studied in connection with spherical trigonometry, and applied to astronomy and navigation, a chapter which belongs distinctly to college curricula. Spherical geometry as a pure science, that is, as a special case of elliptic geometry, is not an appropriate subject for secondary teaching, and the subject is not so treated in American texts.

But it should be emphasized that the greatest economy of time would result from a radical departure from the customary formalism of the subject as now presented in most American high schools. To cite an instance of this, it is not sufficient to say that two vertically opposite angles are equal because they are supplements of the same angle adjacent to both. One must needs go through the ritual of drawing two intersecting lines,

lettering the angles by means of five letters, and solemnly repeating steps and "authorities" in a column, or worse, in two columns. This is supposed to be in the interest of perfection of logic. Consider Euclid's first proposition, for example. It may not be known to some readers that this is a problem, viz.: "To construct an equilateral triangle on a straight line." He is very careful to state precisely how the two circles are to be drawn, to distinguish between the lines *AB* and *BA* as radii, and to give authorities for every step of the proof that his triangle is equilateral. But he wholly omits to prove that his circles meet at all, nor does he give any "axiom" or "postulate" covering this case!

A change of spirit from the excessive formalism of geomtery to something more humane and in keeping with other subjects in the secondary curriculum will be brought about, if at all, through the medium of new textbooks. It goes without saying that it will meet with violent opposition from many who feel that plane geometry ought to be a sheet-anchor in the stormy sea of loose argument which prevails in politics and too often even in natural science. But the fact is that at present the mode of presentation of the subject gives it an unearthly aspect, a thing so wholly apart from the world of real existence that good people boast of their unfamiliarity with it.

But once the idea is fully grasped that the chief aim of geometry teaching is to afford an analytical insight into the world of sense, the secondary curriculum will afford no better subject than a year of plane and solid geometry.

THE SLIDE RULE AS A SUBJECT OF REGULAR CLASS INSTRUCTION IN MATHEMATICS

By WILLIAM E. BRECKENRIDGE
Teachers College, New York City

In the past year regular class instruction in the use of the slide rule has increased rapidly. During the war it was found that the slide rule was a very efficient calculating instrument on account of its rapidity of operation and the ease with which it could be carried in the pocket. Hence, it became indispensable in the artillery and in other branches of the service where computation was required. The supply of rules ordered for the use of the U. S. Government was so great that one manufacturing firm was 20,000 rules behind its orders. It was also found that the rule was a very interesting and convenient subject of study for soldiers in hospitals. This study was promoted by the publication under Government supervision of a monograph on the slide rule called Unit Course No. 1 in Mathematics, the purpose of which was to teach the use of the slide rule by very simple exercises with a minimum of theory. After the close of the war, colleges that had taught the rule for military purposes continued its use in classes of trigonometry. This practice was followed by secondary schools as rapidly as teachers came to understand the use of the rule. Today in many colleges and secondary schools the slide rule is regularly taught in connection with the work in trigonometry. This gives the student an excellent application of the theory of logarithms and equips him with the power to solve instantly nearly all problems in mechanics, physics, and chemistry, with a great saving of time.

It would be an advantage if the rule could be taught in the mathematics classes early enough to be used in physics. For this purpose and because of the intrinsic interest in the work itself, it seems that the plan of several high schools for using the slide rule in the first year is admirable.

In the second or third year of the junior high school, where a review of mathematics is desired, the slide rule has been found very attractive to the students and very useful in computation and in checking numerical work.

The recommendation of the National Committee on Mathematical Requirements that the slide rule be used in junior high schools and in senior high schools has helped to stimulate interest in this instrument.

In shop mathematics and in engineering courses, the rule will continue to be used extensively as in the past.

Recently I observed a class in trigonometry in one of the large high schools of New York City. After solving by the usual method a triangle where two sides and the included angle were given, the results were tested by the slide rule. If an error appeared, it was easily located by the rule. If no error appeared, then the regular check formula of trigonometry was applied. Without the use of the slide rule an error could only have been detected by reviewing the entire computation.

Official recognition of the value of the slide rule was given last June in New York State when the Niagara Falls High School asked and secured permission for its students to use the slide rule in the State Regents' Examination in trigonometry. Since the examiners require all logarithmic work to be shown on the answer paper, the permission to use the slide rule as a check seems perfectly safe.

The time necessary to master the slide rule varies according to the ability of the student. The calculation of per cents may be learned in a few minutes. To secure speed and accuracy on all operations it is desirable to use four or five recitation periods.

Students should be required to buy their own slide rules. Where this is not possible, the school may buy rules enough for one class and use them in the classroom where drawing instruments are used.

With the proper help from textbooks and manuals it is perfectly possible for a teacher to teach himself and his class the use of the rule. The enthusiasm with which this instrument is received by the students is evidence that we shall soon see the slide rule taught regularly in every college and scondary school.

I shall be glad to give any desired information to teachers regarding equipment for teaching the slide rule.

NEWS AND NOTES

During the summer months a drive for new members for the National Council of Teachers of Mathematics was conducted in every State in the Union. In so far as possible it was extended to all summer schools attended by secondary school teachers. While it is impossible to give exact figures the increased number of members gives evidence that the drive was effective.

State representatives have been appointed throughout the Nation. They are undertaking an extensive drive this fall. Every reader of the MATHEMATICS TEACHER is urged to help in this drive by urging his fellow teachers to become members of the Council.

The February meeting of the National Council of Teachers of Mathematics will be held in Chicago during the meeting of the Department of Superintendents. Watch the MATHEMATICS TEACHER for the program.

The next regular annual meeting of the Central Association of Science and Mathematic Teachers will be held in St. Louis in November. The mathematic section of this association ranks among the largest and most effective organizations of teachers of high school mathematics in America.

PROFESSOR J. H. MINNICK, President of the National Council of Teachers of Mathematics, has been elected Dean of the School of Education, University of Pennsylvania.

THE Mathematics Club of St. Louis entertained the members of the National Committee on Mathematical Requirements at a joint meeting of the two groups in April, 1921. About two hundred fifty teachers and other educators were present.

Prof. J. W. Young, of Dartmouth, Chairman of the National Committee, emphasized the need of leisure for the teacher of mathematics, insisting that the day a teacher ceases to be a stu-

dent he or she also ceases to be a real teacher. Miss Vevia Blair, of Horace Mann School, New York City, gave some very interesting statistics upon mathematics in its relation to the transference of training. Mr. Raleigh Schorling of the Lincoln School, New York, briefly and humorously told of the place of mathematics in the lives of boys and girls and was followed by Mr. Downey of Boston, who dwelt upon the need of making the subject attractive to the pupils and thus inspiring interest. Prof. C. N. Moore told very interestingly of the value of mathematics in every day life. He showed with quite definite statistics the dependence of civilization upon the results which mathematics has unsealed to the world in general.

Prof. David Eugene Smith was the chief speaker of the evening and was given the last position on the program. His address on mathematics and citizenship was inspiring, his humor catching, his scholarship manifested itself in his grasp of human life and the expression of his thought.

Credit for the success of the banquet-program is due to President Davis, to Miss Meta Eitzen, Secretary-Treasurer, and to Miss Weeks and Mr. Ammerman, members of the Executive Committee of the Mathematics' Club. (William J. Ryan, S. J.)

In order to affiliate more closely with the National Council of Teachers of Mathematics, the Mathematics Section of the Inland Empire Teachers' Association was reorganized at the annual meeting at Spokane, March 30th to April 1st, as the Inland Empire Council of Teachers of Mathematics. It was voted to become a group member of the National Council and also to urge teachers of mathematics in the Inland Empire to take out individual membership in the National Council. The Inland Empire is a term used to indicate Eastern Oregon, Eastern Washington, Northern Idaho and Western Montana, but the Inland Empire Teachers' Association has grown in the last twenty-five years so that it includes many teachers from all portions of these four northwest states.

Professor W. C. Eells of Whitman College was elected first President of the new organization and Miss Jessie Oldt at North Central High School, Spokane, Secretary.

An organization committee composed of Miss Gertrude Kaye of Spokane, Miss Erma Wylder of Almira, and Mr. Russell L. C. Butsch of Sprague, is working out the details of organization. It is planned to have three permanent committees studying present mathematical problems. One of these, with Miss Kaye as Chairman, will undertake, this year, the investigation of the teaching of correlated mathematics in the high schools of the Inland Empire.

At the Spokane meeting the History, Purpose and Value of the National Council of the Teachers of Mathematics, was discussed by Miss Wylder, and a helpful address outlining some of the activities of the Inland Empire Council of Teachers of English was given by Professor W. R. Davis of Whitman College. Miss Kaye then presented the following motion, which was passed unanimously:

"Resolved, That we endorse the plans and purposes of the newly organized National Council of Mathematics' Teachers and that we organize ourselves into the Inland Empire Council of Mathematics Teachers."

The greater part of the meeting was spent in the discussion of the report on College Entrance Requirements in Mathematics of the National Committee on Mathematical Requirements. This was discussed from the standpoint of colleges and universities by Professor Colpitts of Washington State College; from the standpoint of the larger high schools by Miss Kate Bell, Lewis & Clark High School, Spokane; from the standpoint of the smaller high schools by Mr. Russell L. C. Butsch, Sprague; and from the standpoint of the State Superintendent by Mr. Edwin Twitmyer, State High School Inspector of Washington.

The sentiment of the meeting, on the whole, was very favorable to the changes and standards recommended by the National Committee.

THE National Committee on Mathematical Requirements on September 5th held its last meeting under its present form of organization. One phase of its work has come to an end. The manuscript of a summary of the final report of the committee

has been sent to the U. S. Bureau of Education for publication. This summary, which will constitute a bulletin of some eighty pages, virtually presents the first part of the complete report. It contains the following chapters:

I. A Brief Survey of the Report.
II. Aims of Mathematical Instruction—General Principles.
III. Mathematics for Years Seven, Eight and Nine.
IV. Mathematics for Years Ten, Eleven and Twelve.
V. College Entrance Requirements in Mathematics.
VI. List of Propositions in Plane and Solid Geometry.
VII. The Function Concept in Elementary Mathematics.
VIII. Terms and Symbols in Elementary Mathematics.

And also a brief synopsis of the remaining chapters of the complete report. It is expected that this summary will appear late in November or early in December.

It was the original intention of the committee to publish its complete report also through the U. S. Bureau of Education. It was found, however, that this would involve a delay of two or three years in view of the fact that it would have been necessary for the Bureau of Education to issue the report in parts extending over a considerable period of time. It is hoped at present that sufficient funds will be obtainable to print the report during the winter and to distribute it free of charge to all who are sufficiently interested to ask for it. The complete report will constitute a volume of about five hundred pages. In addition to the chapters listed in the summary, it will contain an account of a number of investigations instituted by the committee. Among these may be mentioned:

The Present Status of Disciplinary Values in Education; A Critical Study of the Correlation Method Applied to Grades; Mathematical Curricula in Foreign Countries; Mathematics in Experimental Schools; The Use of Mental Tests in the Teaching of Mathematics; The Training of Teachers of Mathematics.

There will also be included an extensive bibliography on the teaching of mathematics.

In closing this phase of its work, the committee desires to extend its most cordial thanks to all the individuals and organ-

izations that have helped. The response secured by the committee to its appeal for assistance in solving the many problems facing it has been extremely enthusiastic and gratifying. This leads the committee to look forward to the future optimistically. The real work for which the committee was appointed may be said to begin with the publication of its report rather than to end with it. Continued enthusiastic activity on the part of all individuals and organizations concerned with the teaching of mathematics is needed over a period of many years to put the recommendations of the committee into effect, to test their validity and to modify them in ways that experience shows to be desirable. In order to be of assistance in this direction, the committee hopes to be able to maintain an office with a certain amount of clerical help during the next few years so that it may continue to act as a clearing house for ideas and to stimulate the discussion of problems relating to the teaching of mathematics among the nearly one hundred organizations that have in the past been actively cooperating with the committee.

The recommendations of the National Committee have been made the object of classroom presentation and discussion at a large number of summer schools throughout the country this summer. Indeed some of the most prominent institutions have built the work in mathematics intended for the preparation of teachers around the various preliminary reports of the National Committee.

Professor E. R. Hedrick of the University of Missouri lectured before a number of institutions on behalf of the National Committee from June 20th to August 9th. The institutions visited were

The University of Texas; the University of Oklahoma; the University of Nebraska; the State Normal Schools at Peru and Kearney, Nebraska; the University of Chicago; the University of Iowa; Iowa State Teachers' College; the University of Michigan; Northwestern University.

Professor Hedrick was enthusiastically received at all of these institutions.

BOOK REVIEWS

Mathematics and Life Activities. Suggestions for Students of Mathematics, prepared by the Department of Mathematics of Brown University. Providence, R. I., 1921, pp. 7.

Teachers of mathematics in secondary schools as well as colleges owe a debt of gratitude to Brown University for having published this pamphlet, filled as it is with excellent counsel to students who are planning their work, not merely in mathematics as a science, but in mathematics as an aid to success in other lines. It suggests the benefits to be derived from the study of mathematics, and this with entire freedom from undue claims and in a language that does not require a technical educational vocabulary for its comprehension. It suggests the methods which are helpful to students, not merely in the college but in the secondary schoool as well. And finally, it considers those occupations which require mathematics for their successful pursuit, and in connection with each of these occupations it sets forth the courses that may be taken with the greatest profit.

Although they are intended primarily for college students, so important are the suggestions as to methods of study that, with the permission of the Department they are here reproduced:

Suggestions as to Methods of Studying Mathematics

"Every student has his own peculiar mental characteristics and must find out for himself what methods of study, in his own case, yield the best results. The following suggestions, however, have proved their general value and should be carefully considered.

"1. *Concentrate on your work during your time for study.* The power of focusing attention for an extended period of time is one of the most valuable things that college can contribute, and this habit will do much to assure your success either in the college or in the world. .

"2. ***Do not allow avoidable interruptions by yourself or others.*** A little will-power enables a student to ignore minor disturb-

izations that have helped. The response secured by the committee to its appeal for assistance in solving the many problems facing it has been extremely enthusiastic and gratifying. This leads the committee to look forward to the future optimistically. The real work for which the committee was appointed may be said to begin with the publication of its report rather than to end with it. Continued enthusiastic activity on the part of all individuals and organizations concerned with the teaching of mathematics is needed over a period of many years to put the recommendations of the committee into effect, to test their validity and to modify them in ways that experience shows to be desirable. In order to be of assistance in this direction, the committee hopes to be able to maintain an office with a certain amount of clerical help during the next few years so that it may continue to act as a clearing house for ideas and to stimulate the discussion of problems relating to the teaching of mathematics among the nearly one hundred organizations that have in the past been actively cooperating with the committee.

The recommendations of the National Committee have been made the object of classroom presentation and discussion at a large number of summer schools throughout the country this summer. Indeed some of the most prominent institutions have built the work in mathematics intended for the preparation of teachers around the various preliminary reports of the National Committee.

Professor E. R. Hedrick of the University of Missouri lectured before a number of institutions on behalf of the National Committee from June 20th to August 9th. The institutions visited were

The University of Texas; the University of Oklahoma; the University of Nebraska; the State Normal Schools at Peru and Kearney, Nebraska; the University of Chicago; the University of Iowa; Iowa State Teachers' College; the University of Michigan; Northwestern University.

Professor Hedrick was enthusiastically received at all of these institutions.

Mathematics and Life Activities. Suggestions for Students of Mathematics, prepared by the Department of Mathematics of Brown University. Providence, R. I., 1921, pp. 7.

Teachers of mathematics in secondary schools as well as colleges owe a debt of gratitude to Brown University for having published this pamphlet, filled as it is with excellent counsel to students who are planning their work, not merely in mathematics as a science, but in mathematics as an aid to success in other lines. It suggests the benefits to be derived from the study of mathematics, and this with entire freedom from undue claims and in a language that does not require a technical educational vocabulary for its comprehension. It suggests the methods which are helpful to students, not merely in the college but in the secondary schoool as well. And finally, it considers those occupations which require mathematics for their successful pursuit, and in connection with each of these occupations it sets forth the courses that may be taken with the greatest profit.

Although they are intended primarily for college students, so important are the suggestions as to methods of study that, with the permission of the Department they are here reproduced:

Suggestions as to Methods of Studying Mathematics

"Every student has his own peculiar mental characteristics and must find out for himself what methods of study, in his own case, yield the best results. The following suggestions, however, have proved their general value and should be carefully considered.

"1. *Concentrate on your work during your time for study.* The power of focusing attention for an extended period of time is one of the most valuable things that college can contribute, and this habit will do much to assure your success either in the college or in the world. .

"2. *Do not allow avoidable interruptions by yourself or others.* A little will-power enables a student to ignore minor disturb-

ances and distractions. Education is a part of your business in life and in the acquiring of it you should form business habits.

"3. *Feel sure of success.* Few intelligent and industrious students are unable to comprehend fundamental processes of mathematical reasoning. If you are discouraged, consult with an instructor as to the cause of your difficulties and the remedy for your lack of success.

"4. *Reflect as you read.* Learning by rote contributes little to educational development, and the habitual performance of tasks in a mechanical way leads to intellectual atrophy.

"5. *Be mentally alert, active and aggressive.* Apply the principles of the lessons to concrete problems; such application is the test of understanding and requires effort and initiative.

"6. *Enjoy overcoming obstacles.* It is worth while in itself, gives power for surmounting new obstacles, and is good preparation for success in life. Love of difficulty is essential to high attainment.

"7. *Work alone at least part of the time.* Discuss the subject with fellow students, but think it over and do detailed work by yourself, both before and after such discussion.

"8. *Attend class regularly.* Prepare each lesson regularly and systematically. In the logical development of the subject each lesson plays its part so that a lack of understanding of earlier work is a bar to progress.

"9. *Devote enough time to study.* The length and difficulty of the assignments and the severity of the marking are intended to be such that a student of average ability and preparation, studying each lesson from an hour and a half to two hours, will receive the grade C. A student deficient in either ability or preparation must be content with less than this grade unless he devotes more time to study."

The bulletin should be in the hands of all who have to do with elective courses in our high schools.

David Eugene Smith

Examples in Differential and Integral Calculus, with Answers. By the late C. S. Jackson. London, Longmans, Green and Co., 1921. Pp. viii + 142. Price 10s. 6d.

When there came, with the World War, the demand for the more intensive training of officers in the Royal Military Academy at Woolwich, and for the extending of this training to thousands of new men in the service, there was assumed, and enthusiastically assumed, by one of the instructors, a burden far in excess of his strength. He had long been a faithful, enthusiastic, successful teacher, and in the early days of the war he fell—a victim to his sense of duty. This man was C. S. Jackson, one of the most lovable men in the teaching profession of England, a fine mathematician, a loyal friend, and a gentleman in the truest sense. He closed his classroom door late one afternoon after a hard day's work, stepped out of the building, and fell dead. Such was one of the many thousands of tragedies of the war—as real a sacrifice as any made in the trenches of France and Flanders.

Mr. Jackson left a manuscript which has been edited by Mr. W. M. Roberts and which now appears in book form. It is fortunate that it is published at this time, for never before has so much attention been given to a proper, human introduction to the elementary calculus. England has had several worthy handbooks of this general class. She has made several attempts at an easy approach, and she has made many other attempts (perhaps more than any other country) at a scholarly approach, but it is doubtful if any of her teachers have hitherto presented such a satisfactory combination of modern educational thought, of simplicity of attack, and of scholarly treatment as is here set forth.

The work is divided into two parts—the first relating to the differential calculus (pages 1-53), and the second to the integral calculus (pages 54-120). It properly begins with tangents and slopes, defines dy/dx as a rate of increase, and proceeds at once to easy maxima and minima. It adds a little more theory, takes up Newton's method of roots, and then considers more difficult cases of maxima and minima. Some idea of the further scope of the work may be derived from a mention of

such topics as velocity, errors and rates, examples on maps, explanations of planimeters and integraphs, and a study of mean volumes, of center of pressure, and of the pendulum.

Any one who is experimenting with the calculus in secondary schools will do well to have a copy of this work in hand, and, indeed, to place it in the hands of the students as a source book from which to draw the problems for class work. The chief doubt as to its use as a textbook lies in the fact that the publishers find that war prices must still be maintained. This fact will unquestionably be adverse to the sale that we would all like to see in this country.

DAVID EUGENE SMITH

The New Methods in Arithmetic. By EDWARD L. THORNDIKE. Rand McNally & Co. Pp. 260.

So many books on the teaching of arithmetic are either a mere collection of discussions of miscellaneous topics more or less related or of platform addresses revised and edited for purposes of publication. Usually such books are worth one reading but not serious study. The present book is written for "the working teacher or student in a normal school seeking direct help in understanding the newer methods and using them under conditions of classroom instruction." It is an authoritative discussion in non-technical language of the principles underlying the teaching of arithmetic. Both the teacher and the student interested in better teaching of arithmetic will want to read and re-read, to study and study again, this illuminating book.

The main purpose of the author is to state and discuss the general principles "that guide the teacher in choosing topics, in arousing and utilizing interest, in securing understanding of the science of arithmetic, ability to compute and ability to apply arithmetic to the problems of the real world, and in organizing arithmetic into a series of instructive experiences and activities." At the same time he shows how to apply these general principles "and also all the helpful conclusions that classroom experience and scientific studies of the learning process have reached, to every detail of the teaching of arithmetic."

Every chapter contains illustrations of the newer methods taken from the author's own textbooks in arithmetic. (The Thorndike Arithmetics, a three-book series, Rand McNally & Co.) In all, there are about sixty-five pages of these illustrations. Herein lies one important advantage of this book over any other book on the teaching of arithmetic. Every principle is amply illustrated by lessons and pages taken from these arithmetics written in accordance with what the author calls the newer methods. The illustrations are so satisfying that the reader will want to turn to the arithmetics from which they are taken to see them in their actual setting. In addition to this each chapter has at the close from two to six pages of very helpful exercises providing abundant opportunity for useful study.

The author is continually speaking by way of contrast about the newer methods and the older methods. Some readers will disagree with him in characterizing as older methods many plans of proceedure still in wide usage. For example, considerable time is spent by many teachers in developing with the pupils the fundamental processes with integers in terms of units, tens, hundreds, etc. The newer methods at the appropriate time say, "This is the way to do it," and without any developmental explanation give in mechanical order the necessary steps to get the answer. Then some satisfactory proof that the result is right is given. This gives confidence that the procedure is correct, and thus the process is rationalized, according to the newer methods. At no time do the newer methods neglect the thought side of arithmetic. In fact the newer methods claim to teach the automatic phases of arithmetic in the most effective and economic way and thus have more time to concentrate greater effort where real thinking is actually needed. One evidence that the author is not altogether wrong is the fact that a few of the latest arithmetics have made copious use of the outstanding features of the Thorndike arithmetics.

There is need in the field of secondary mathematics for a similar book on the new methods in algebra and one on the new methods in geometry. Such books are needed to bring about a more united effort and a more apparent harmony in the teaching of secondary mathematics. However, teachers of secondary mathematics, and of junior high school mathematics, too, will be

benefited in reading *The New Methods in Arithmetic,* especially the chapters entitled "Reality," "Interest," "Theory and Explanations," "Habit Formation and Drills," and "Solving Problems." Any teacher of mathematics who studies this book will be encouraged to justify his methods and material on other than traditional grounds.

EDGAR C. HINKLE

Chicago Normal College